Provided for non-commercial research and education use. Not for reproduction, distribution or commercial use.



This article was published in an Elsevier journal. The attached copy is furnished to the author for non-commercial research and education use, including for instruction at the author's institution, sharing with colleagues and providing to institution administration.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

http://www.elsevier.com/copyright



Available online at www.sciencedirect.com





Journal of Magnetism and Magnetic Materials 320 (2008) 1541-1543

www.elsevier.com/locate/jmmm

# Effects of grain size distribution on coercivity and permeability of ferromagnets

Desheng Xue, Guozhi Chai\*, Xiling Li, Xiaolong Fan

Key Laboratory for Magnetism and Magnetic Materials of MOE, Lanzhou University, 222 Tianshui Road, Lanzhou, Gansu 730000, PRChina

Received 12 November 2007 Available online 4 January 2008

#### Abstract

Grain size dependence of coercivity and permeability (GSDCP) theory is extended to include grain size distribution in ferromagnets. It is found that the experimental data do not agree with the GSDCP theory on the transition location of different grain size ranges (The GSDCP theory has three different grain size ranges for different magnetization processes.). Correspondingly, including the grain size distribution the GSDCP theory fits the experimental data very well. These results prove that the grain size distribution indeed affects the magnetic properties of nanocrystalline ferromagnets.

© 2008 Elsevier B.V. All rights reserved.

PACS: 75.30.Gw; 75.75+a

Keywords: Random anisotropy; Size distribution; Nanocrystalline ferromagnet; Grain size dependence

#### 1. Introduction

The advent of miniaturized devices has led to extending material science to nanomaterials. However, with the reduction of size, material properties will be tremendously changed. For magnetic material, coercivity and permeability as functions of average grain size  $D_{\text{eff}}$  can be described by the grain size dependence of coercivity and permeability (GSDCP) theory [1–5]. The GSDCP theory shows that the coercivity agrees with the  $D^6$  law for particle sizes smaller than the exchange length, the constant law for particle sizes between the exchange length and domain wall width, and the 1/D law for particle sizes larger than the domain wall width. As the changes of magnetic properties with grain size can be well described by this theory, GSDCP theory becomes the theoretical foundation of magnetic materials preparation.

However, the non-uniform grain size in a real case results in a derivation from the GSDCP; especially near the connections of the different laws even when an average grain size was used. Silva et al. [6] found a lognormal

\*Corresponding author. Fax: +869318912726.

E-mail address: chaigzh2006@lzu.cn (G. Chai).

distribution in realistic grain sizes, obtained from TEM images. This means near the connections the grains have different laws in the real case. An average grain size causes the GSDCP to differ from the experimental data near the connections. The distribution of the grain size should be considered in calculating the coercivity and permeability by using GSDCP. In this paper, coercivity and permeability in different ranges with GSDCP were calculated by using a lognormal grain size distribution, and the result was discussed.

### 2. Theoretical model

The calculation of coercivity and permeability are based on the GSDCP theory. The effect of the grain size distribution on magnetic properties of this system is derived by introducing the lognormal distribution function in the GSDCP theory, which can be given by

$$H_{c}(D_{eff}) = \int_{0}^{L_{ex}} H_{c}^{1}(D)f(D) dD + \int_{L_{ex}}^{\delta_{B}} H_{c}^{2}(D)f(D) dD + \int_{\delta_{B}}^{\infty} H_{c}^{3}(D)f(D) dD, \qquad (1a)$$

<sup>0304-8853/\$ -</sup> see front matter © 2008 Elsevier B.V. All rights reserved. doi:10.1016/j.jmmm.2008.01.004

D. Xue et al. / Journal of Magnetism and Magnetic Materials 320 (2008) 1541-1543

$$\mu_{i}(D_{eff}) = \int_{0}^{L_{ex}} \mu_{i}^{1}(D)f(D) dD + \int_{L_{ex}}^{\delta_{B}} \mu_{i}^{2}(D)f(D) dD + \int_{\delta_{B}}^{\infty} \mu_{i}^{3}(D)f(D) dD,$$
(1b)

where  $D_{\text{eff}}$  is mean grain size, D is realistic grain size,  $L_{\text{ex}} = (A/K_1)^{1/2}$ , A is the exchange constant,  $K_1$  is magnetocrystalline anisotropy constant,  $\delta_{\text{B}} = \pi L_{\text{ex}} = \pi (A/K_1)^{1/2}$ is the domain wall width and f(D) is the lognormal distribution function given by

$$f(D) = \frac{1}{\sqrt{2\pi\sigma}D} \exp\left(-\frac{\ln^2(D/D_{\text{eff}})}{2\sigma^2}\right),$$
(2)

where  $\sigma$  is the standard deviation,  $H_c^n$  and  $\mu_i^n$  (n = 1,2,3) are the functions of the coercivity and permeability of the materials in the GSDCP theory [1].  $H_c^n$  and  $\mu_i^n$  (n = 1,2,3)should be written as

$$\begin{cases} H_{\rm c}^{1} = p_{\rm c} \frac{K_{\rm eff}}{M_{\rm S}} = p_{\rm c} \frac{K_{\rm 1}^{4} D^{6}}{M_{\rm s} A^{3}}, & \mu_{\rm i}^{1} = p_{\mu} \frac{M_{\rm s}^{2}}{K_{\rm eff}} = p_{\mu} \frac{M_{\rm s}^{2} A^{3}}{K_{\rm 1}^{4} D^{6}}, & (D < L_{\rm ex}) \\ H_{\rm c}^{2} = p_{\rm c} \frac{K_{\rm 1}}{M_{\rm S}}, & \mu_{\rm c}^{2} = p_{\mu} \frac{M_{\rm s}^{2}}{K_{\rm 1}}, & (L_{\rm ex} < D < \delta_{\rm B}) \\ H_{\rm c}^{3} = p_{\rm c} \frac{\sqrt{AK_{\rm 1}}}{M_{\rm S} D}, & \mu_{\rm c}^{3} = p_{\mu} \frac{M_{\rm s}^{2} D}{\sqrt{AK_{\rm 1}}}, & (L_{\rm ex} > \delta_{\rm B}), \end{cases} \end{cases}$$
(3)

where  $M_s$  is average saturation magnetization of the materials,  $p_c$  and  $p_{\mu}$  are dimensionless factors close to



Fig. 1. Lognormal distribution curves of  $D_{\rm eff} = 20$  for different values of  $\sigma$ .

unity and  $K_{\text{eff}}$  is the effective anisotropy constant which can be written as  $K_{\text{eff}} = K_1 (D/L_{\text{ex}})^6$  [1].

### 3. Results and discussion

In order to compare with the GSDCP theory results, we use the same magnetic parameters as the GSDCP theory [1], where  $A = 10^{-11} \text{ J/m}$ ,  $K_1 = 8 \times 10^3 \text{ J/m}^3$ ,  $L_{\rm ex} = (A/K_1)^{1/2} = 35 \text{ nm}, p_{\rm c} = 0.13, p_{\mu} = 0.5 \text{ for } D_{\rm eff} < L_{\rm ex}, p_{\rm c} = 0.64, p_{\mu} = 0.33 \text{ for } L_{\rm ex} < D_{\rm eff} < \delta_{\rm B}, p_{\rm c} = 2.6, p_{\mu} = 0.05$ for  $D_{\rm eff} > \delta_{\rm B}$ . The standard deviation  $\sigma$  can be determined from the properties of the materials (R.W. Chantrell analyzed the standard deviation from magnetization curve of the materials [7]). In this work, median grain diameters are considered as mean grain diameters. Then,  $\sigma$  can be determined by Eq. (1a), while coercivities and mean grain diameters are known quantities. From Fig. 1 we can see that almost all grain sizes are below  $L_{ex}$ , when mean grain sizes below 20 nm no matter how large the  $\sigma$  is. To make sure that coercivity cannot be affected by different rules in GSDCP, the experimental data whose mean grain sizes are <20 nm are chosen to analyze  $\sigma$ . The coercivity can be described by the same function with  $D_{\rm eff}$  for  $D_{\rm eff} < 20$  nm. The two data which are presented in Fig. 2 are chosen to calculate  $\sigma$ . The results show that two  $\sigma$  are both 0.1. So the lognormal distribution for  $\sigma = 0.1$  is used to calculate the coercivity and the permeability of  $FeCu_{0-1}Nb_{1-3}(SiB)_{22.5}$ magnets.

Fig. 2 shows the experimental data (marked with point), the result of GSDCP (marked with dashed line) theory and our result (marked with real line). As shown in Fig. 2, the experimental data do not agree with the GSDCP theory for  $20 \text{ nm} < D_{\text{eff}} < 150 \text{ nm}$ : GSDCP theory result is larger than the experimental data of coercivity for  $D_{\text{eff}}$  near the exchange length and the domain wall width. We explain the result with three different cases: (1) For mean grain size slightly less than  $L_{\text{ex}}$ , GSDCP calculates the coercivity with the  $D^6$  law. But as shown in Fig. 3 there are some grain sizes larger than  $L_{\text{ex}}$  in a real particle system which has a grain size distribution. So coercivities of these grains should be calculated with the constant law. For these grains values of the  $D^6$  law are larger than of the constant



Fig. 2. Coercivity (left) and permeability (right) as functions of average particle size  $D_{\text{eff}}$ .

D. Xue et al. / Journal of Magnetism and Magnetic Materials 320 (2008) 1541-1543



Fig. 3. Distribution width of  $\sigma = 0.1$  for different values of  $D_{\text{eff}}$  compared with exchange length  $L_{\text{ex}}$  and domain wall width  $\delta_{\text{B}}$ .

law. (2) As shown in Fig. 3, there are some grain sizes less than  $L_{\rm ex}$  or larger than  $\delta_{\rm B}$  for  $L_{\rm ex} < D_{\rm eff} < \delta_{\rm B}$ . The constant value is larger than their real values. This causes the GSDCP result to be larger than the experimental data in this part. (3) Similarly, the values of the  $D^{-1}$  law are larger than the constant law for grain sizes less than  $\delta_{\rm B}$ . So the theoretical values are larger than the experimental data for  $D_{\rm eff}$  slightly larger than  $\delta_{\rm B}$ .

In comparison, it is found that by considering the grain size distribution our result fits the experimental data very well as shown in Fig. 2. The reason is that each grain size dependence in the system is considered in our calculation rather than using an average grain size to calculate the property of the entire system. Comparing our result with the GSDCP theory's result, it is obvious that small deviations can result in larger coercivity for  $20 \text{ nm} < D_{\text{eff}} < 150 \text{ nm}$ . If we want to obtain high coercivity grains, grain size should be controlled between  $L_{\text{ex}}$  and  $\delta_{\text{B}}$  and made uniform as much as possible.

# 4. Conclusion

This work provides an effective method for studying coercivity and permeability of ferromagnets, whose mean grain size is close to the exchange length or the domain wall width. The result shows that including grain size distribution, GSDCP fits experimental data very well even if the grain size is close to the exchange length and domain wall width. This result also proves that the grain size distribution affects coercivity and permeability of particle ferromagnets.

## Acknowledgments

This work is supported by NSFC (Grant No. 10374038 and No. 50671046) and the Trans-Century Training Program Foundation for the Talent of MOE, PRChina.

#### References

- [1] G. Herzer, IEEE Trans. Magn. 26 (1990) 1397.
- [2] R. Alben, J.J. Backer, M.C. Chi, J. Appl. Phys. 49 (1978) 1653.
- [3] G. Herzer, IEEE Trans. Magn. 25 (1989) 3327.
- [4] G. Herzer, Scr. Metal. Mater. 33 (1995) 1741.
- [5] G. Herzer, J. Magn. Magn. Mater 294 (2005) 99.
- [6] F.C.S. da Silva, M. Knobel, D. Ugarte, D. Zanchet, IEEE Trans. Magn. 36 (2000) 3430.
- [7] R.W. Chantrell, J. Popplewell, S.W. Charles, IEEE Trans. Magn. MAG-14 (1978) 975.